

# Neutrino mass operators of dimension up to nine in two-Higgs-doublet model

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## Abstract

We study higher-dimensional neutrino mass operators in a low energy theory that contains a second Higgs doublet, the two Higgs doublet model. The operators are relevant to underlying theories in which the lowest dimension-five mass operators would not be induced. We list the independent operators with dimension up to nine with the help of Young tableau. Also listed are the lowest dimension-seven operators that involve gauge bosons and violate the lepton number by two units. We briefly mention some of possible phenomenological implications.

PACS: 14.60.Pq, 14.60.Lm, 12.60.Fr

Keywords: neutrino mass, seesaw, effective field theory, two Higgs doublets

DOI information: 10.1016/j.physletb.2011.03.025

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# 1 Introduction

The tiny neutrino mass can be accommodated at low energies by nonrenormalizable, higher-dimensional mass operators. With the lepton fields as established in the standard model (SM) and the Higgs fields assumed to be a doublet, such operators first appear at dimension five [1]:

$$\mathcal{O}_{xy}^{\alpha\beta} = \overline{F_{Lx}^C} \tilde{H}_\alpha^* \tilde{H}_\beta^\dagger F_{Ly}, \quad \mathcal{P}_{xy}^{\alpha\beta} = \overline{F_{Lx}^C} \tilde{F}_{Ly}^* \tilde{H}_\beta^\dagger H_\alpha. \quad (1)$$

Here  $H_\alpha$  is the  $\alpha$ -th Higgs doublet with hypercharge  $Y = +1$ , and  $F_{Lx}$  is the  $x$ -th left-handed leptonic doublet with  $Y = -1$ . A tilde denotes the complex-conjugated field that transforms under  $SU(2)_L$  exactly as the original one, e.g.,  $\tilde{F}_L = i\sigma^2 F_L^*$ , while the superscript  $C$  denotes charge conjugation with the convention  $F_L^C = (F_L)^C$ .

Both operators  $\mathcal{O}$  and  $\mathcal{P}$  break the lepton number by two units. When the neutral components of the scalar doublets develop a vacuum expectation value (VEV),  $\mathcal{O}$  generates a mass for neutrinos that is inversely proportional to the energy scale  $\Lambda$  of some underlying theory responsible for the operator. Although the operator  $\mathcal{P}$  does not generate a mass but involves interactions amongst leptons and scalars of different charge, it may arise from the same mechanism that induces  $\mathcal{O}$  due to the similar structure. With a single Higgs doublet as in SM, the operator  $\mathcal{O}$  is unique while  $\mathcal{P}$  does not exist.

It is interesting to realize that the unique operator  $\mathcal{O}$  in SM may be written in three apparently different ways [2]. This amounts to forming a singlet in three ways out of four factors of the two half-isospin fields, and suggests its possible origin from three types of seesaw mechanisms [3, 4, 5]. A phenomenological issue with those mechanisms is that the energy scale  $\Lambda$  is so high that it would not be possible to detect any other effects pertained to the origin of neutrino mass. From the viewpoint of effective field theory, the scale may be lowered if the mass is induced not from a dimension-five operator but from those of even higher dimensions. It is conceivable that there will be more and more mechanisms that can induce a mass operator as its dimension increases, see Refs. [6, 7] for some recent examples. However, it has been established recently that the mass operator at each higher dimension is always unique [8]. This implies that as far as the neutrino mass is concerned different mechanisms are completely equivalent. But with a lowered scale, it becomes possible to distinguish them through other effects.

In this work we will address the neutrino mass operators in an effective field theory that contains two Higgs doublets. Although the two Higgs doublet model (2HDM) is interesting in itself, the main motivation comes from supersymmetry which is a leading candidate for physics beyond SM and is under examination at high energy colliders. It would be also tempting to see how those higher-dimensional operators are induced in a supersymmetric framework. We will show that with two Higgs doublets the operators are no more unique but increase quickly in number with their dimension. We will list all mass operators of dimension up to nine as well as related dimension-seven operators involving the SM gauge fields. We will also discuss briefly some of the phenomenological implications of these operators at low energies.

## 2 Mass operators up to dimension nine

We assume that the low energy theory contains the SM fields and an additional Higgs doublet that also develops a VEV. The neutrinos can only have a Majorana mass in this case. We are interested in the high-dimensional operators that can yield a neutrino mass (called  $\mathcal{O}$ -type) when the Higgs fields assume their VEV's, as well as those that do not give a mass but have a similar structure ( $\mathcal{P}$ -type). We can therefore restrict ourselves to the two-lepton sector that violates the lepton number by two units. The relevant fields are the lepton doublet  $F_L$ , the two Higgs doublets  $H_1$  and  $H_2$  plus their properly complex-conjugated fields which also transform as a doublet under  $SU(2)_L$ :

$$a = \overline{F_L^C} (-1), b = F_L (-1), c = H_1 (+1), d = H_2 (-1), e = \tilde{H}_2 (+1), f = \tilde{H}_1 (-1), \quad (2)$$

where the number in parentheses indicates hypercharge. Our notation is such that we always use column spinors in isospin space though  $\overline{F_L^C}$  is a row spinor in Dirac space and should appear on the left of  $F_L$  to form an appropriate Dirac bilinear. The lepton generation index is generally inessential and can be easily recovered when necessary. We note the following features that are useful to exhaust all possibilities. First, since the pair  $ab$  appears once, there are two more factors of  $c$  or  $e$  than  $d$  or  $f$  to balance hypercharge. The dimension of mass operators is thus  $2n + 5$ , where  $n$  denotes the number of copies of  $d$  or  $f$ . Second, the occurrence of  $c$  may be replaced by  $e$  if this yields a different and nonvanishing result, and similarly with  $d$  and  $f$ . Finally, the SM case is recovered by the identifications  $e = c$  and  $d = f$ .

With an even number of fields with nonzero isospin one may imagine to form higher isospin products before building a singlet out of them. But this is unnecessary when all the fields are in the fundamental representation (spinor for short) of  $SU(2)$ : all isospin invariants of a given mass dimension can be exhausted by first forming singlets from any two spinors and then multiplying them. This is the group-theoretical reason that the three types of seesaws reduce to the unique dimension-five Weinberg operator  $\mathcal{O}$  in SM [2] and that its higher-dimensional generalizations are also unique at each dimension [8].

The above point can be best seen in the tensor method in terms of Young tableau. For  $SU(2)$  a Young tableau has at most two rows, and each column with two rows is a separate invariant. This is especially convenient when only spinors appear, because in that case each box represents an individual field and a two-row column is an antisymmetric, invariant product of the two spinors involved. This has a few immediate consequences. First, there can be no bare mass term from  $\overline{F_L^C} F_L$  even if  $F_L$  had a zero hypercharge. Second, denoting a spinor by its index in the box, we have the basic relation:

$$\begin{array}{|c|c|} \hline i & m \\ \hline j & n \\ \hline \end{array} - \begin{array}{|c|c|} \hline i & m \\ \hline n & j \\ \hline \end{array} = \begin{array}{|c|c|} \hline i & j \\ \hline m & n \\ \hline \end{array} \quad (3)$$

which is equivalent to the relation  $(i, j, m, n = 1, 2)$

$$\epsilon_{ij}\epsilon_{mn} - \epsilon_{in}\epsilon_{mj} = \epsilon_{im}\epsilon_{jn}. \quad (4)$$

Applied to the dimension-five Weinberg operators in eq (1), we have

$$\mathcal{O}_{xy}^{\alpha\beta} - \mathcal{O}_{yx}^{\alpha\beta} = \mathcal{P}_{xy}^{\alpha\beta}, \quad (5)$$

which means that only one group of dimension-five operators (type  $\mathcal{O}$ ) listed in Ref [1] are actually independent. (Be careful not to mix the generation indices with the spinor indices.) More generally, putting spinors directly in boxes we have

$$\begin{array}{|c|c|} \hline a & \kappa \\ \hline b & \sigma \\ \hline \end{array} \mathcal{Y} = \begin{array}{|c|c|} \hline a & b \\ \hline \kappa & \sigma \\ \hline \end{array} \mathcal{Y} - \begin{array}{|c|c|} \hline a & b \\ \hline \sigma & \kappa \\ \hline \end{array} \mathcal{Y} \quad (6)$$

where  $\mathcal{Y}$  is any Young tableau. Namely, the  $\mathcal{P}$ -type operators that contain as a factor an invariant formed out of  $a, b$  are linear compositions of the  $\mathcal{O}$ -type operators. By making a complete list of all mass operators (of type- $\mathcal{O}$ ), all non-mass operators (of type- $\mathcal{P}$ ) with a similar structure are automatically covered. In the language of Young tableau, we will never put  $a, b$  in the same column.

It is easy to figure out all dimension-five operators since  $d, f$  cannot appear while  $c/e$  appears twice. They are

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & c \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & e \\ \hline \end{array} \quad (7)$$

plus those obtained by  $c \leftrightarrow e$ , or

$$S_5 = (a, c)_0(b, c)_0, \quad T_5 = (a, c)_0(b, e)_0, \\ \bar{S}_5 = S_5|_{c \leftrightarrow e}, \quad \bar{T}_5 = T_5|_{c \leftrightarrow e}, \quad (8)$$

where the subscript 0 denotes an isospin invariant formed by antisymmetrizing the fields inside the parentheses; for instance, denoting the upper (lower) component of a spinor by a subscript plus (minus) sign, we have  $\sqrt{2}(a, c)_0 = a_+c_- - a_-c_+$ . Since  $a$  and  $b$  are essentially the same field, the list of operators may be further reduced. To see this clearly, we reserve the lepton generation index by putting  $a = \overline{F_{Lx}^C}$ ,  $b = F_{Ly}$ . Then,

$$2\bar{T}_5^{xy} = (\overline{v_{Lx}^C}e_- - \overline{f_{Lx}^C}e_+)(v_{Ly}c_- - f_{Ly}c_+) = (\overline{v_{Ly}^C}c_- - \overline{f_{Ly}^C}c_+)(v_{Lx}e_- - f_{Lx}e_+) = 2T_5^{yx}, \quad (9)$$

where  $\overline{\psi_i^C} \psi_j = \overline{\psi_j^C} \psi_i$  is used. We can thus choose  $S_5, \bar{S}_5, T_5$  as the complete and independent list of dimension-five operators.

At dimension seven, the operators contain three copies of  $c$  or  $e$  and one copy of  $d$  or  $f$ , and can be classified as  $S : c^3d, T : c^2ed$ , plus those obtained by  $c \leftrightarrow e$ , or  $d \leftrightarrow f$ , or both interchanges. The first one is easy to write down:

$$S_7 = (a, c)_0(b, c)_0(d, c)_0. \quad (10)$$

For the second one, there are following possibilities to distribute the spinors in the boxes of a  $2 \times 3$  Young tableau:

$$\begin{array}{c} \begin{array}{|c|c|c|} \hline a & b & d \\ \hline c & c & e \\ \hline \end{array} \\ \text{1st} \end{array} \quad \begin{array}{c} \begin{array}{|c|c|c|} \hline a & b & d \\ \hline c & e & c \\ \hline \end{array} \\ \text{2nd} \end{array} \quad \begin{array}{c} \begin{array}{|c|c|c|} \hline a & b & e \\ \hline c & d & c \\ \hline \end{array} \\ \text{3rd} \end{array} \quad \begin{array}{c} \begin{array}{|c|c|c|} \hline a & b & d \\ \hline e & c & c \\ \hline \end{array} \\ \text{4th} \end{array} \quad \begin{array}{c} \begin{array}{|c|c|c|} \hline a & b & e \\ \hline d & c & c \\ \hline \end{array} \\ \text{5th} \end{array} \quad (11)$$

But the basic relation in eq (3) implies

$$1\text{st} - 2\text{nd} + 3\text{rd} = 0, \quad 1\text{st} - 4\text{th} + 5\text{th} = 0, \quad (12)$$

which eliminate two operators. We choose the 1st, 3rd, and 5th ones to be independent:

$$T_7^1 = (a, c)_0(b, c)_0(d, e)_0, T_7^2 = (a, c)_0(b, d)_0(e, c)_0, T_7^3 = (a, d)_0(b, c)_0(e, c)_0. \quad (13)$$

But for the same reason as for  $\bar{T}_5$ ,  $T_7^3$  is covered by  $T_7^2$  when the lepton generation indices are reserved, and may thus be excluded as redundant. The remaining operators are obtained by interchanges:

$$\begin{aligned} \bar{S}_7 &= S_7|_{c \leftrightarrow e}, \hat{S}_7 = S_7|_{d \leftrightarrow f}, \tilde{S}_7 = S_7|_{c \leftrightarrow e, d \leftrightarrow f}; \\ \bar{T}_7^{1,2} &= T_7^{1,2}|_{c \leftrightarrow e}, \hat{T}_7^{1,2} = T_7^{1,2}|_{d \leftrightarrow f}, \tilde{T}_7^{1,2} = T_7^{1,2}|_{c \leftrightarrow e, d \leftrightarrow f}. \end{aligned} \quad (14)$$

There are altogether 12 operators at dimension seven.

The dimension-nine operators contain four copies of  $c$  or  $e$  and two copies of  $d$  or  $f$ , which are classified as  $S : c^4 d^2$ ,  $T : c^4 d f$ ,  $U : c^3 e d^2$ ,  $V : c^3 e d f$ ,  $W : c^2 e^2 d^2$ ,  $X : c^2 e^2 d f$ , plus those obtained by interchange  $c \leftrightarrow e$ , or  $d \leftrightarrow f$ , or both. We continue to denote an operator obtained by  $c \leftrightarrow e$  with a bar, that by  $d \leftrightarrow f$  with a hat, and the one by both  $c \leftrightarrow e$  and  $d \leftrightarrow f$  with a tilde. It is easy to write down  $S$  and  $T$ :

$$S_9 = S_7(d, c)_0, T_9 = \hat{S}_7(d, c)_0. \quad (15)$$

And there are four more operators that are obtained by interchanges:

$$\bar{S}_9, \hat{S}_9, \tilde{S}_9; \bar{T}_9. \quad (16)$$

The  $U$  operators have one more factor of  $cd$  than  $T_7$ . Deleting the redundant one associated with  $T_7^3$ , we have

$$U_9^{1,2} = T_7^{1,2}(d, c)_0. \quad (17)$$

The additional operators obtained by interchanges are also independent

$$\bar{U}_9^{1,2}, \hat{U}_9^{1,2}, \tilde{U}_9^{1,2}. \quad (18)$$

It is possible to distribute in nine ways the spinors of  $V$  in a  $2 \times 4$  Young tableau, but only four of them yield independent operators. Five Young tableaux are obtained from those in eq (11) by attaching an additional column of  $(f, c)_0$ , thus giving the three independent operators:

$$V_9^{1,2,3} = T_7^{1,2,3}(f, c)_0. \quad (19)$$

Another three tableaux are obtained from the above by  $d \leftrightarrow f$ :  $\hat{V}_9^{1,2,3}$ , and the ninth one corresponds to

$$V_9^0 = (a, c)_0(b, c)_0(d, f)_0(e, c)_0. \quad (20)$$

Similar to  $T_5^3$  and  $T_7^3$ ,  $V_9^3$  can be deleted from the list. Furthermore, the basic relation (3) implies that

$$\hat{V}_9^1 - V_9^1 = V_9^2 - \hat{V}_9^2 = V_9^3 - \hat{V}_9^3 = -V_9^0, \quad (21)$$

so that we can keep  $V_9^{0,1,2}$  in the list while excluding  $\hat{V}_9^{1,2,3}$  as redundant. Finally, there are three more operators obtained from interchange  $c \leftrightarrow e$ :

$$\bar{V}_9^{0,1,2}. \quad (22)$$

A similar (but slightly different) analysis applies to the operators  $W$ , which have four independent forms

$$W_9^{1,2} = T_7^{1,2}(d, e)_0, \bar{W}_9^{1,2}, \quad (23)$$

plus four more by interchange  $d \leftrightarrow f$ :

$$\hat{W}_9^{1,2}, \tilde{W}_9^{1,2}. \quad (24)$$

Finally we come to the symmetric case of  $X$  that has the most possible Young tableaux (18 in total). The basic relation (3) removes ten of them as redundant and the symmetry in the lepton fields deletes another three, leaving us with five independent operators:

$$\begin{aligned} X_9^{1,2} &= T_7^{1,2}(f, e)_0, \bar{X}_9^1 \\ X_9^A &= (a, c)_0(b, e)_0(d, f)_0(c, e)_0, \\ X_9^S &= (a, d)_0(b, f)_0(c, e)_0(c, e)_0, \end{aligned} \quad (25)$$

where  $X_9^S$  ( $X_9^A$ ) is (anti)symmetric in  $c \leftrightarrow e$  and  $d \leftrightarrow f$  respectively when the lepton generation indices are ignored. There are altogether 33 dimension-nine mass operators.

### 3 Adding gauge bosons

The underlying physics that produces the higher-dimensional neutrino mass operators in the last section may also induce lepton-number violating interactions with gauge bosons. In this section we continue to work in the two-lepton sector and list the lowest dimension-seven operators with gauge bosons that are built upon the dimension-five mass operators. The gauge fields may enter in two ways, either through gauge covariant derivatives or through field strength tensors. The first case amounts to introducing new Lorentz vector fields that have the same quantum numbers under the SM gauge group as the original fields,  $a$ ,  $b$ ,  $c$ ,  $e$ . The second case requires that those original fields must be built into a hypercharge-neutral, isospin-triplet or -singlet form that couples to the field strength tensors of  $SU(2)_L$  and  $U(1)_Y$  respectively.

We start with the operators containing the gauge covariant derivative

$$D_\mu = \partial_\mu - ig_2 \frac{1}{2} \sigma^a W_\mu^a \mp ig_1 \frac{1}{2} B_\mu, \quad (26)$$

where the minus (plus) sign applies to the fields  $c$ ,  $e$  ( $a$ ,  $b$ ), and  $W_\mu^a$  and  $B_\mu$  are the gauge fields with gauge couplings  $g_{2,1}$ . Distributing two factors of  $D_\mu$  to any two of the four fields in the

mass operators  $S_5$  and  $T_5$  yields

$$\begin{aligned} J^{1,\dots,6} &= (D_\mu a, D^\mu c)_0(b, c)_0, (D_\mu a, c)_0(D^\mu b, c)_0, (D_\mu a, c)_0(b, D^\mu c)_0, \\ &\quad (a, D_\mu c)_0(b, D^\mu c)_0, (a, D_\mu c)_0(D^\mu b, c)_0, (a, c)_0(D_\mu b, D^\mu c)_0; \end{aligned} \quad (27)$$

$$\begin{aligned} K^{1,\dots,6} &= (D_\mu a, D^\mu c)_0(b, e)_0, (D_\mu a, c)_0(D^\mu b, e)_0, (D_\mu a, c)_0(b, D^\mu e)_0, \\ &\quad (a, D_\mu c)_0(b, D^\mu e)_0, (a, D_\mu c)_0(D^\mu b, e)_0, (a, c)_0(D_\mu b, D^\mu e)_0; \end{aligned} \quad (28)$$

plus  $\bar{J}^{1,\dots,6}$  and  $\bar{K}^{1,\dots,6}$  that are obtained by  $c \leftrightarrow e$ . Since the gauge covariant derivative does not spoil the relation  $\bar{\psi}_i^C \psi_j = \bar{\psi}_j^C \psi_i$ , we can exclude some of the operators as redundant as we did with  $\bar{T}_5$ . Reserving the lepton generation indices and denoting the upper (lower) component of a gauge covariant derivative also by a subscript plus (minus) sign, we have, for instance,  $a_{x+}(D_\mu b_y)_- = (D_\mu a_y)_- b_{x+}$  using our notations in eq (2). It should be reminded that no integration by parts can be legitimately used here; instead, the relation  $\bar{\psi}_i^C \psi_j = \bar{\psi}_j^C \psi_i$  is sufficient. Some inspection then shows that  $J_{xy}^6 = J_{yx}^1$ ,  $J_{xy}^5 = J_{yx}^3$  and similarly for  $\bar{J}$ . Since the  $K$  operators involve simultaneously  $c$  and  $e$  fields, a stronger reduction of operators becomes possible, namely,  $\bar{K}_{xy}^{1,2,3,4,5,6} = K_{yx}^{6,2,5,4,3,1}$ . The complete and independent operators can thus be chosen to be

$$J^{1,\dots,4}, \bar{J}^{1,\dots,4}, K^{1,\dots,6}. \quad (29)$$

To construct dimension-seven operators involving gauge field strength tensors, the Lorentz indices of the tensors must be contracted by Dirac matrices. This means that a  $\sigma^{\mu\nu}$  should be sandwiched between the lepton fields  $a$  and  $b$ , which fits well with their chiralities. Consider first the coupling of  $B_{\mu\nu}$  to a singlet formed from  $abce$ . The only difference to the dimension-five mass operator  $T_5$  is to insert a  $\sigma^{\mu\nu}$  between  $a$  and  $b$ :

$$M(B) = (a, c)_0 \sigma^{\mu\nu} (b, e)_0 B_{\mu\nu}. \quad (30)$$

Note that  $\bar{M}(B)$ , which is again obtained from  $M(B)$  by the interchange  $c \leftrightarrow e$ , is not independent since when attaching the lepton generation indices to  $a$  and  $b$  we have  $\bar{M}_{xy}(B) = -M_{yx}(B)$  upon using  $\bar{\psi}_x^C \sigma_{\mu\nu} \psi_y = -\bar{\psi}_y^C \sigma_{\mu\nu} \psi_x$ . It is not necessary either to consider the case where  $a, b$  lie in the same column of a tableau since the basic relation (3) is not disturbed by the Lorentz structure. Similarly, the counterparts of  $S_5$  and  $\bar{S}_5$  are

$$L(B) = (a, c)_0 \sigma^{\mu\nu} (b, c)_0 B_{\mu\nu}, \bar{L}(B). \quad (31)$$

In contrast to the above, the  $SU(2)_L$  gauge field strength  $W_{\mu\nu}^a$  being a triplet must couple to the triplet states of  $abce$  to become a singlet. There are apparently nine possibilities for  $abce$  to form a triplet state: one pair of spinors in a singlet and the other in a triplet (six in total), or both pairs in a triplet multiplied into a triplet (three in total). But only three of them are independent as we show below. We note first of all that there are four possible ways to form a state with  $I_3 = +1$ , which do not necessarily have a definite total isospin  $I$ :

$$w = a_- b_+ c_+ e_+, x = a_+ b_- c_+ e_+, y = a_+ b_+ c_- e_+, z = a_+ b_+ c_+ e_-. \quad (32)$$



But symmetry requires that the  $I_3 = +1$  state of a triplet ( $I = 1$ ) formed from four spinors be a difference of the above quantities, and thus there can only be three independent states with  $I_3 = +1$  belonging to three triplets. (The fourth  $I_3 = +1$  state has  $I = 2$ , and all this is consistent with isospin composition indeed.) For instance, the  $I_3 = +1$  state formed from  $ab$  in a singlet and  $ce$  in a triplet is  $(w - x)/\sqrt{2}$ , while the  $I_3 = +1$  state with all of  $ab$ ,  $ce$ , and  $abce$  in a triplet is given by  $(w + x - y - z)/2$ .

To write isospin-1 states formed with two isospin-half ones, it is convenient to use the row spinor. We denote by a check the combined action of tilde and dagger on the isospin space, which transforms a column spinor to a row spinor in the complex representation. For instance,

$$\check{b} = \tilde{b}^\dagger = (f_L, -v_L), \quad (33)$$

can form a singlet with  $c$  and the gauge field strength,  $\check{b}W_{\mu\nu}c$ , where  $W_{\mu\nu} \equiv \frac{1}{2}\sigma^a W_{\mu\nu}^a$ . The complete and independent couplings of  $W_{\mu\nu}^a$  to  $abce$  are therefore as follows:

$$M^1(W) = (a, \sigma^{\mu\nu}b)_0 \check{c}W_{\mu\nu}e, \quad M^2(W) = (a, c)_0 \sigma^{\mu\nu} \check{b}W_{\mu\nu}e, \quad \bar{M}^2(W). \quad (34)$$

$\bar{M}^2(W)$  is independent of  $M^2(W)$  since  $c$  and  $e$  are now at inequivalent places in contrast to  $M(B)$ . On the other hand,  $\bar{M}^1(W) = M^1(W)$  because  $\check{c}W_{\mu\nu}e = \check{e}W_{\mu\nu}c$ . This is in accord with the above symmetry arguments. The operators with two  $c$  or two  $e$  are

$$L^1(W) = (a, \sigma^{\mu\nu}b)_0 \check{c}W_{\mu\nu}c, \quad L^2(W) = (a, c)_0 \sigma^{\mu\nu} \check{b}W_{\mu\nu}c, \quad \bar{L}^1(W), \quad \bar{L}^2(W). \quad (35)$$

To summarize, the complete and independent dimension-seven operators involving the gauge field tensors are  $L(B)$ ,  $\bar{L}(B)$ ,  $M(B)$ ,  $L^1(W)$ ,  $L^2(W)$ ,  $\bar{L}^1(W)$ ,  $\bar{L}^2(W)$ ,  $M^1(W)$ ,  $M^2(W)$  and  $\bar{M}^2(W)$ .

## 4 Discussion

The effective operators that we have written down in the last two sections involve various lepton-number violating interactions of multi-Higgs and gauge bosons, which may have rich phenomenological implications. But to make a complete analysis, we should include some other operators at a similar dimension, in particular those involving four-fermions, that violate the lepton number by two units. Operators involving four and six fermions in SM were analyzed in Refs. [9, 10] for inducing neutrino mass at the loop level and their phenomenology explored in [10]. The neutrino mass operators with two Higgs doublets were symbolically written down in Ref. [7] from hypercharge balance but no attempt was made to complete their isospin structures. Instead, possible underlying models were suggested that could induce a specific dimension-seven operator via seesaw, together with radiative mechanisms.

In this concluding section, we discuss briefly some interesting interactions contained in the operators listed in the last sections, while leaving a more complete phenomenological analysis for the future work. Assume both Higgs doublets develop VEV's which are generally complex with phases  $u_{1,2}$ ,

$$\langle H_1^0 \rangle = \frac{v}{\sqrt{2}} u_1 c_\beta, \quad \langle H_2^0 \rangle = \frac{v}{\sqrt{2}} u_2 s_\beta, \quad (36)$$



where  $v = 246$  GeV and  $c_\beta = \cos \beta$ ,  $s_\beta = \sin \beta$ . The would-be Goldstone bosons  $G^{\pm,0}$  and physical scalars  $H^\pm$ ,  $A^0$ ,  $R_\alpha$  ( $\alpha = 1, 2$ ) are related to the original fields by unitary transformations:

$$\begin{pmatrix} G^- \\ H^- \end{pmatrix} = \begin{pmatrix} u_2^* s_\beta & -u_1 c_\beta \\ u_1^* c_\beta & u_2 s_\beta \end{pmatrix} \begin{pmatrix} H_2^- \\ H_1^- \end{pmatrix}, \quad (37)$$

$$\begin{pmatrix} iI_\alpha \\ R_\alpha \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} u_\alpha^* & -u_\alpha \\ u_\alpha^* & u_\alpha \end{pmatrix} \begin{pmatrix} H_\alpha^0 \\ H_\alpha^{0*} \end{pmatrix}, \quad (38)$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} s_\beta & -c_\beta \\ c_\beta & s_\beta \end{pmatrix} \begin{pmatrix} I_2 \\ I_1 \end{pmatrix}. \quad (39)$$

When CP is conserved,  $A^0$  is a pseudoscalar while  $R_{1,2}$  are scalars whose mixing is determined by the scalar potential.

Attaching the lepton generation indices, the operator  $T_5$ , for instance, contains a term

$$-\frac{1}{4} u_1 u_2^* v^2 c_\beta s_\beta \overline{\nu_{Lx}^C} \nu_{Ly}, \quad (40)$$

which gives neutrino mass after incorporating a coefficient matrix in generations. The phases  $u_1 u_2^*$  can be removed by redefinition of fields, but will reappear in other terms of  $T^5$  that involve the Higgs scalars and a lepton pair. These interactions are relatively hard to explore since the dominant decays of the scalars generically depend on the details of the underlying theory. Furthermore, as we discussed in Introduction, to have any chance at all to discover the mass generation mechanism, the mass should be generated from operators of a high enough dimension so that the relevant physics scale could be lowered. A promising scenario would be that the mass operators are generated at, say, dimension nine, while the lepton-number violating operators involving gauge fields are generated at dimension seven by the same physics through tree-level or one-loop effects. For these interactions we can say something more certain since we know how the gauge bosons interact with the SM fermions. We therefore will concentrate on them in what follows.

Consider first the operators involving gauge field tensors, for instance,  $M(B)$ . In addition to terms involving scalars, it contains the dipole interactions with the  $Z$  boson and photon  $A$ :

$$+\frac{1}{4} u_1 u_2^* v^2 c_\beta s_\beta (s_W Z_{\mu\nu} - c_W A_{\mu\nu}) \overline{\nu_{Lx}^C} \sigma^{\mu\nu} \nu_{Ly}, \quad (41)$$

with  $c_W = \cos \theta_W$  and  $s_W = \sin \theta_W$ . A similar dipole term also appears in  $M^2(W)$ :

$$-\frac{1}{4\sqrt{2}} u_1 u_2^* v^2 c_\beta s_\beta (c_W Z_{\mu\nu} + s_W A_{\mu\nu}) \overline{\nu_{Lx}^C} \sigma^{\mu\nu} \nu_{Ly}. \quad (42)$$

These terms are also contained in the corresponding  $L$  operators and barred operators except for different factors of  $u_{1,2}$ ,  $c_\beta$  and  $s_\beta$ . For any given value of  $\beta$  there are always operators that are not suppressed by its triangular functions. We assign a common coefficient  $eC_{d.m.}$  to (the sum) of those operators while ignoring factors of order one. While Majorana neutrinos have no dipole moments due to CPT invariance, they can accommodate transition dipole moments between

different neutrinos [11]. Roughly speaking, the upper bounds on the latter are about  $10^{-10}\mu_B$  or weaker from laboratory experiments [12] and about  $10^{-12}\mu_B$  from astrophysical arguments on energy loss in stars [13]. Here  $\mu_B = e/(2m_e)$  is the Bohr magneton. They translate into a bound on the coefficient of the operators:

$$C_{\text{d.m.}} \lesssim \frac{10^{-10} \text{ or } 10^{-12}}{m_e v^2}, \text{ i.e., } C_{\text{d.m.}} \lesssim (6.7 \text{ or } 31 \text{ TeV})^{-3}. \quad (43)$$

For the operators involving gauge covariant derivatives, the most interesting interaction is the one that contributes to the neutrinoless double beta decay,

$$\begin{aligned} -J^{2,4} &= J^3 = \frac{1}{2} m_W^2 u_1^2 c_\beta^2 \mathcal{Q}_{xy} + \dots, \\ -\bar{J}^{2,4} &= \bar{J}^3 = \frac{1}{2} m_W^2 (u_2^*)^2 s_\beta^2 \mathcal{Q}_{xy} + \dots, \\ K^{2,4} &= -K^{3,5} = \frac{1}{2} m_W^2 u_1 u_2^* c_\beta s_\beta \mathcal{Q}_{xy} + \dots, \end{aligned} \quad (44)$$

while  $J^{1,6}$ ,  $K^{1,6}$ ,  $\bar{J}^{1,6}$  do not contain the operator  $\mathcal{Q}_{xy} = \overline{f_{Lx}^C} f_{Ly} W^{+\mu} W_\mu^+$ . Here  $m_W = \frac{1}{2} g_2 v$  is the  $W^\pm$  boson mass. We assign a common coefficient  $C_{xy}$  to (the sum) of these operators. Barring exceptional cancellation, their contribution to the subprocess  $W^- W^- \rightarrow ee$ ,  $\sim C_{ee} m_W^2$ , should not exceed the usual one via the exchange of light active neutrinos, which is experimentally bounded and given by  $\sim m_{ee}/q^2$ . Here  $m_{ee} = \sum_j m_j U_{ej}^2$  with  $m_j$  being the mass of the neutrino  $\nu_j$  and  $U$  the leptonic mixing matrix, and  $q \sim (50 \sim 100) \text{ MeV}$  is the momentum transfer. The upper bounds on  $m_{ee}$  [14] then imply that

$$|C_{ee}| \lesssim \frac{|m_{ee}|}{|q^2| m_W^2} \sim (5 \text{ TeV})^{-3}, \quad (45)$$

where we assume for order of magnitude estimation,  $|m_{ee}| \sim 0.5 \text{ eV}$  and  $q \sim 100 \text{ MeV}$ .

The operators displayed in the last section also contain other interactions involving multiple scalars and gauge bosons, or modify the SM interactions. We leave this more complete phenomenological analysis for the future work which should better include the effects of multiple-fermion operators with a comparable dimension.

## Acknowledgement

This work is supported in part by the grants NSFC-10775074, NSFC-10975078, NSFC-11025525, and the 973 Program 2010CB833000.

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